

## NEGATION AND CONTRADICTION

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The problems of the meaning and function of negation are disentangled from ontological issues with which they have been long entangled. The question of the function of negation is the crucial issue separating relevant and paraconsistent logics from classical theories. The function is illuminated by considering the inferential role of contradictions, contradiction being parasitic on negation. Three basic modellings emerge: a cancellation model, which leads towards connexivism, an explosion model, appropriate to classical and intuitionistic theories, and a constraint model, which includes relevant theories. These three modellings have been seriously confused in the modern literature: untangling them helps motivate the main themes advanced concerning traditional negation and natural negation. Firstly, the dominant traditional view, except around scholastic times when the explosion view was in ascendancy, has been the cancellation view, so that the mainstream negation of much of traditional logic is distinctively nonclassical. Secondly, the primary negation determinable of natural negation is relevant negation. In order to picture relevant negation the traditional idea of negation as otherthanness is progressively refined, to nonexclusive restricted otherthanness. Several pictures result, a reversal picture, a debate model, a record cabinet (or files of the universe) model which help explain relevant negation. Two appendices are attached, one on negation in Hegel and the Marxist tradition, the other on Wittgenstein's treatment of negation and contradiction.

### 1. THE PHILOSOPHICAL CENTRALITY OF NEGATION, AND THE HISTORICAL ENTANGLEMENT OF NEGATION WITH ONTOLOGICAL ISSUES.

Negation is a fundamental, but ill-understood, ill-explained and much disputed notion across a wide philosophical spectrum. It is not only the central notion in recent and momentum-gathering disputes between classically-inclined logicians and alternative people (called by rougher classical types 'deviants'); it is also, for instance, a crucial notion in much Buddhist philosophy, such

as theories of meaning, relation and cognition (see, e.g., Shaw, Matilal, and references therein). But in order 'to make sense of the use of negation in Buddhist philosophy in general, one needs to venture outside the perspective of the standard (i.e. classical) notion of negation' (Matilal, p.2). As well, negation, along with its derivate nothingness, is a key notion in modern European philosophy, for instance, in the modern tradition Sartre considers himself to belong to, from Hegel through Husserl and Heidegger (see Sartre, p.6 ff); again the negations involved are, almost invariably, nonclassical.

However very many of the problems, accounted problems of negation in the literature, are not really problems of negation simpliciter at all but are rather problems of nonexistence, which arise from the alleged riddle of not-being or nonexistence; of how it can be truly said, as it sometimes is, that A does not exist (or that A is not) when the truth of the statement implies that A does exist (that A is). Such are central issues in Greek philosophy, from Parmenides through Plato's *Sophist*; such are the main worries of late nineteenth and early twentieth century traditional logicians over negative judgements and negative terms; such are many of the problems in existentialism over not-being and nothingness. As to the second consider, for example, the main logical difficulties Joseph finds with negative judgements:

Judgement... refers to the existent, whose manner of being is as we conceive. But the real is positive: it only exists by being something, not by being nothing. A negative judgement declares what is not, and how can this express it as it is?(<sup>1</sup>)

As to the third, consider for instance Sartre's (somewhat devious but similar) argument to the objective existence of non-being and nothing (p.5) (<sup>2</sup>). All these moves and their difficulties, are based in one way or another upon the Ontological Assumption, according to which what is a subject of true discourse must exist, a thoroughly fallacious assumption whose manifold defects have already been exposed (in Routley, [22], Chapter 1).

These perennial "problems" persist in contemporary logical theory. Much of Russell's problem with negation, for example, is again an ontological one:

if the sun is not shining there is not a fact *sun-not-shining* which is affirmed by the true statement "the sun is not shining" (p.520).

For if there were, it would exist, yet how can such negativity exist? It cannot according to Russell, in sharp contrast to existentialists, who grasp the other horn of the dilemma the Ontological Assumption generates from negative facts. Fortunately, Russell (erroneously) thinks, negation is eliminable: '...

'not' is unnecessary for a complete description of the world' (p.520). He makes two gestures towards showing this, one psychological and quite unsatisfactory - 'what is happening [in the belief that "the sun is not shining"] is that I am inhibiting the impulses generated by the belief that the sun is shining - and one logical - "not" is eliminated from our fundamental apparatus' through 'the positive predicates "true" and "false"'. But this is no elimination without cheating, namely reclassifying the negative predicate 'false' as a positive predicate (which makes negation itself positive, since it can then be positively defined!). For, otherwise, as falsity involves negation, 'false' being defined commonly (for a very wide range of logical theories) in terms of 'true' and 'not', the account Russell presupposes, that a statement not-A is true iff A is false, is circular, and begs the question (cf. Quine, p.84). Similarly the psychological account is circular at bottom, since 'inhibit' is a negative verb (derived using the negative prefix 'in'). In fact the indefinability of negation in various positive logics is readily demonstrated (see, e.g., Goddard and Routley, chapter 5). Negation cannot be eliminated without cheating: nor can it be dispensed with without very serious impoverishment of discourse, as Griss's attempt to construct a negationless mathematics and von Dantzig's an affirmative mathematics have revealed (see, e.g., Fraenkel and Bar-Hillel, p.239 ff).

## 2. IT IS AS REGARDS NEGATION THAT RELEVANT AND PARAconsistent LOGICS DIVERGE FUNDAMENTALLY FROM CLASSICAL LOGIC.

Even with the ontological problems duly disposed of, many problems remain, especially as to the meaning and function of negation. Some of these problems are grappled with in what follows, especially the problems of characterising, picturing and modelling negations of relevant (and also paraconsistent) logics, and, what overlaps these, negations of natural language. A *relevant* logic can be characterized, approximately for present purposes, as a logic whose pure sentential part conforms to Belnap's weak relevance requirement, namely that there is no thesis of the form  $A \rightarrow B$ , (i.e. that A implies or entails that B) where A and B do not share a variable. (A *standard* relevant logic is one that conservatively extends the first degree system FD, the first degree of system E and R and very many other relevant logics. Much of the discussion of relevant logic that follows is conceived in terms of standard systems, though some points will apply more widely).

The fundamental divergence of relevant (and paraconsistent) logics from classical logic is as to negation, its logical and inferential behaviour. Indeed at the first degree stage (where no nested implications occur), *relevant and classical logic differ just over negation* (see Routley, Meyer, Plumwood and Brady, *Relevant Logics and Their Rivals* [23], hereafter RLR, Chapter 2).

Negation is accordingly, really the crucial notion for the choice of logical theory, as also for comparisons of logical theories, for appreciation of the varieties and character of competing theories of entailment, and so on.

Parasitic on negation is contradiction. A contradictory situation is one where both  $B$  and  $\sim B$  (it is not the case that  $B$ ) hold for some  $B$ . An explicit contradiction is a statement of the form  $B$  and  $\sim B$ . A statement  $C$  is contradictory, it is often said, if it entails both  $B$  and also  $\sim B$  for some  $B$ , etc. Contradiction is always characterized in terms of negation and the logical behaviour of contradictions is dependent on that of negation. Different accounts of negation result not merely in different conceptions of contradiction and of incompatibility, they likewise correspond to different accounts of what constitutes a describable world, what constitutes a logically assessable world. Classical negation restricts such worlds to possible worlds, excluding contradictory and incomplete worlds.

Contradictory situations play a prominent role in world semantics for relevant logics. Most conspicuously, nontrivial contradictory situations are deployed in counterexamples to the harder Lewis paradox of implication, the spread principle,  $\text{ex falso quodlibet}$ ,  $A \wedge \sim A \rightarrow B$  (or in rule form  $A, \sim A \Rightarrow B$ ), which spreads contradictions everywhere and trivializes all contradictory situations. For suppose  $c$  is a nontrivial situation, i.e. not everything holds at  $c$ , but  $c$  is contradictory. Then for some  $A$  and  $B$ ,  $A$  and  $\sim A$  both hold in  $c$  but  $B$  does not. Therefore  $A$  and  $\sim A$  does not entail  $B$ , for this would require that in whatever situation  $A$  and  $\sim A$  hold  $B$  does also.

It is at the same time evident that classical logic and classically-based logics rule out nontrivial inconsistent situations, and so exclude an important class of theories, of *much* philosophical and other interest. More generally, the excluded class is that of paraconsistent theories. The core idea is that a paraconsistent theory is one that contains true contradictions without triviality. It is immediate that paraconsistent logics, logics that can serve as the basis for paraconsistent theories are, rather radically, nonclassical.

Many relevant logics are paraconsistent logics, but not all are. For example, Ackermann's logic  $\Pi^1$  (which has the same theorems as Anderson and Belnap's system  $E$ ) is not a paraconsistent logic; and similarly for almost any relevant logic that has Material Detachment,  $A, \sim A \vee B \rightarrow B$ , as a primitive (or immediate) rule. Systems  $E$  and  $R$  and all their subsystems and many of their extensions are paraconsistent logics. Relevant logics do not however exhaust paraconsistent logics. There are many irrelevant paraconsistent logics, e.g. the main systems of da Costa, the earlier systems of Priest, etc. Relevant and paraconsistent logics thus properly overlap. Standard systems in the overlap are of primary interest in what follows. For these systems a theory of negation, of relevant negation, is especially important.



### 3. BASIC MODELLINGS OF NEGATION IN TERMS OF DIFFERENT RELATIONS OF $\sim A$ TO $A$ .

Theories of negation differ, very obviously, in the roles they allow, or assign to, contradictions. Contradictions may be allowed no inferential role (they imply nothing, except perhaps themselves), a total inferential role (they imply everything), or some limited inferential role (they imply some things, such as their contradictory components, but not others). There are, correspondingly three initial ways to classify theories of negation, in terms of the relation of  $\sim A$  to  $A$ .

1.  $\sim A$  deletes, neutralizes, erases, cancels  $A$  (and similarly, since the relation is symmetrical,  $A$  erases  $\sim A$ ), so that  $\sim A$  together with  $A$  leaves nothing, no content. The conjunction of  $A$  and  $\sim A$  says nothing, so nothing more specific follows. In particular,  $A \wedge \sim A$  does not entail  $A$  and does not entail  $\sim A$ . Accordingly, the *cancellation* (erasure, or neutralization) model leads towards connexivism, a position (much discussed in RLR) distinguished by the following two theses- First, that already cited, that explicit contradictions do not entail their components, and secondly, that  $A$  does not entail  $\sim A$ . The second thesis emerges naturally under the neutralization view, for instance, as follows. Entailment is inclusion of logical content. So, if  $A$  were to entail  $\sim A$ , it would include as part of its content, what neutralizes it,  $\sim A$ , in which event it would entail nothing, having no content. So it is not the case that  $A$  entails  $\sim A$ , that is Aristotle's thesis,  $\sim(A \rightarrow \sim A)$  holds.<sup>(3)</sup>

There is reasonable, but not conclusive, evidence that Aristotle did adhere to Aristotle's thesis. And assuming that he did certainly has great explicative advantages, for example the full theory of the syllogism translates into connexive quantificational logic without loss or qualification (as Angell, and also McCall, has pointed out); the theory of immediate inference also emerges intact (for inferential but not implicational form). Whether or not Aristotle was operating with connexive assumptions, there is a long historical line of logicians and philosophers who have assumed a cancellation picture, from Boethius in medieval times through to Strawson, Körner, and many others in modern times (see RLR). One striking intermediate example is Berkeley, who advances the following claims in his attack on the calculus (*The Analyst*, p. 73):

Nothing is plainer than that no just conclusion can be directly drawn from two inconsistent premises. You may indeed suppose any thing possible: But afterwards you may not suppose anything that destroys what you first supposed: or, if you do, you must begin *de nove*... [When] you... destroy one supposition by another... you may not retain the consequences, or any part of the consequences, of your first supposition so destroyed.

Cancellation views are prevalent in one place where they are particularly damaging, in so-called expositions of Hegel (but there is a basis for this ascription in Hegel himself, as will appear). Given this phenomenon it is not surprising that Hegel's logic has appeared so intractable to commentators. Here, for instance, is what the Marxist logician Havas has to say as regards Hegel's theory:

...the Aristotelian principle of non-contradiction is a general principle of metalogic, which can be said to bring out a necessary condition to be satisfied by all human thought and all of the systems of logic; namely, the condition that it is a logical contradiction, and therefore, a logical mistake to assert both something and its opposite. This is one of the elementary but necessary conditions of sound reasoning, because if one asserts something to be true and, insisting on this assertion, one also asserts that this very assertion is not true, then his assertions will neutralize each other and, in consequence of this, no knowledge will be acquired (p.7).

Apart from being unfaithful to Hegel, who (correctly) says that there is nothing unthinkable about contradictions, thereby repudiating the laws-of-thought myth, and who accepted no such simple neutralization view, the Aristotelian principle is not a metalogical principle concerning the logic of assertion.

The second model for negation is that embodied in contemporary classical and intuitionist logics:

2.  $\sim A$  explodes, or fully implodes,  $A$  (and similarly  $A$  explodes  $\sim A$ ) in such a way that  $\sim A$  together with  $A$  yields everything, total content. The conjunction  $A$  and  $\sim A$  says everything, so everything follows.  $A \wedge \sim A$  entails  $B$ , for arbitrary and irrelevant  $B$ , so the *explosion* (or destruction) model is inevitably paradoxical. The paradoxical character of classical logic for example, can accordingly be obtained with very few further assumptions from the character of its negation.

Under weak, and relatively noncontroversial, conditions on other connectives ( $\sim, \wedge, \vee$ ), the explosion model delivers classical negation, according to which negation  $\sim$ , is evaluated according to the classical semantical rule  $\mathbf{I}(\sim A, a) = 1$  iff  $\mathbf{I}(A, a) \neq 1$ , i.e.  $A$  holds at world  $a$  iff  $A$  does not hold at  $a$ , for every world  $a$  (deployed in the semantic evaluation of entailment). (Under alternative conditions the model yields intuitionistic negation.) Conversely, classical negation, i.e. negation conforming to the classical rule, yields the explosion view, since there is no world where both  $A$  and  $\sim A$  hold but  $B$  does not, whence, on the semantical theory,<sup>(4)</sup>  $A \wedge \sim A \rightarrow B$ . Thus under weak conditions the explosion view is that of classical negation.

Classical negation offers a complete exclusion model of negation, more precisely, an exclusion and exhaustion view: for each world  $a$  and each statement  $A$ ,  $\sim A$  excludes  $A$  from holding in  $a$ , and  $\sim A$  united with  $A$  exhausts  $a$ , one or other must hold in  $a$ . The picture is that commonly offered for the real world (as e.g. in Hospers, p.212) simply relativised to world  $a$ , namely



where the ellipse represents the whole of  $a$ , all statements of  $a$ . 'Not  $A$ ' will cover all territory (of  $a$ ) other than what is covered by ' $A$ ' (p.223).

Quine and many others (e.g. D. Lewis, Copeland) think that classical negation is 'our ordinary' negation and that there is no alternative to it, for any alternative would 'change the subject' from negation. Of course they never *argue* that it is our ordinary negation; they simply *assume* that it is. So it is in Quine's main defence of classical negation, which occurs in a famous passage in 'Deviant Logics' (p.81) where he considers two parties,  $\alpha$  and  $\beta$  say, who proceed as follows:  $\alpha$ , adopting a 'popular extravagance', rejects the law of non-contradiction and accepts  $A$  and  $\sim A$  occasionally.  $\beta$  objects that this 'would vitiate all science' and uses the paradoxes to show that everything would follow so 'forfeiting all distinctions between true and false'. Party  $\alpha$  tries to 'stave' this off by 'compensatory adjustments', by rigging the logic so as to isolate contradictions (in good paraconsistent fashion). In Quine's view,

neither party knows what he is talking about. They think that they are talking about negation, ' $\sim$ ', 'not'; but surely the notation ceased to be recognisable as negation when they took to regarding some conjunctions of the form ' $p, \sim p$ ' as true, and stopped regarding such sentences as implying all others.

Quine's case is however vitiated by being described in a thoroughly incoherent (indeed inconsistent) fashion; for example, party  $\beta$  is described as objecting that 'everything would follow' and as adopting what appears to be the classical view, yet Quine asserts that '*neither* party knows what he is talking about' because *neither* adopts the classical view, having just described one of his disputants as doing so. Nor are Quine's conclusions independently warranted. The paradoxes of strict implication are not built into the ordinary notion of negation, into the particle 'not'. The English negation determinable 'not' is not so determined (as distinct from the classical negation determinate). Quine has failed to observe the distinction, and has done something which entirely begs the question at issue: equated, without any trace of argument, the natural language 'not' with classical negation. Thus what he goes on to

claim has no secure basis:

Here evidently lies the deviant logician's predicament: when he tries to deny the doctrine he only changes the subject (p.81).

There is no predicament: the "deviant" may be trying, with more success than the classicist, to explicate the core notion 'not'. On Quine's viewpoint, no distinct systems can explicate the same (preanalytic) connectives - which is a reduction to absurdity of the position. Moreover, were Quine right no "deviant" could reject the classical doctrine, he would only be changing the subject. Yet elsewhere Quine admits (and has to admit on his theory of unrestricted revisability<sup>(4.1)</sup>) the possibility of rejecting the doctrine (e.g. on p.84, three pages later):

It is hard to face up to the rejection of anything so basic [as classical negation, etc.] If anyone questions the meaningfulness of classical negation, we are tempted to say in defense that the negation of any given closed sentence is *explained* thus: it is true if and only if the given sentence is not true... However our defense here begs the question... [since] we use the same classical 'not'.

*Nor* is it the meaningfulness of classical negation that is at issue: it is its correctness, and its uniqueness. The semantical recipe given in explanation does not separate classical negation from various other negations, e.g. the relevant negation of  $\Pi$ ', which [can] satisfy the same recipe. Accordingly, the recipe does not explain classical negation (without further assumptions, such as a one-world assumption), nor does it show its uniqueness.

Although classical negation is not, unlike connexive negation, a subtraction operation, a taking *away* of something already given, it involves certain subtraction features. By contrast relevant negation does not involve subtraction features;  $\sim A$  does not imply the taking away or elimination of  $A$ , but adds a further condition (although one related to  $A$  by certain constraints);  $\sim A$  does not have entirely dependent status in the way it does classically. These differences are already reflected in the structure of the complete possible worlds of classical logic, as distinct from the worlds of relevant logic. In the classical case when  $\sim A$  is added to a world, quite a bit may have to be taken out of the world, e.g.  $A$  (and what implies it) if it is there, in order to consistencize the world; whereas in the relevant case  $\sim A$  can simply be added without any consistencizing subtractions. More generally, worlds can be simply combined and statements added to worlds without the need to delete anything, *because* what is being added are further conditions, not the taking away of conditions already given. This is the route to a straightforward, and relevant, theory of counterfactuals (in sharp contrast to the irrelevant classically-based theories which presently dominate the literature<sup>(5)</sup>).

3. On the third part of the trichotomous classification,  $\sim A$  neither cancels nor explodes  $A$ , rather  $\sim A$  *constrains* but does not totally control  $A$ . This allows for different positions, including one which will be of especial concern in what follows, namely relevant negation. Equally as "natural" as the cancellation model, and much more natural than the explosion view of contradiction is the relevant model, according to which contradictions have exactly the same sort of inferential status as other types of propositions, that is, they imply some propositions and fail to imply other propositions and are subject to the same laws.

The normal semantical rule for evaluating relevant negation which again is derivable under modest conditions (see RLR, 2.9), is as follows:

$$I(\sim A, a) = 1 \text{ iff } I(A, a^*) \neq 1,$$

i.e.  $\sim A$  holds in world  $a$  iff  $A$  does not hold in world  $a^*$ , the opposite or reverse of  $a$ . The normal rule, which *generalises* the classical rule, differs from the classical rule in the occurrence of function  $*$ , a function which has generated much discussion. A major objective in what follows is further explanation of the  $*$  function. It is not difficult to show that negation so evaluated has the leading properties sought, e.g.  $A$  and  $\sim A$  are suitably independent though nonetheless related;  $A$  and  $\sim A$  may both fail together and differently both may hold together;  $A$  and  $\sim A$  neither cancel nor implode one another.

It is also not too difficult to indicate how requisite allowance for incomplete and inconsistent worlds, both sorts of which are called for in the semantical evaluation of inference, leads to the normal rule for negation. Such was the historical route: given that the paradoxes of strict implication, (1)  $A \wedge \sim A \Rightarrow B$ ; and (2)  $C \Rightarrow D \vee \sim D$ , are indeed paradoxes and false of entailment, and that entailment (at the first degree) amounts to truth (or holding) preservation over worlds, then their semantical evaluation must allow for worlds where  $A$  and  $\sim A$  (strictly  $A \wedge \sim A$ ) hold but  $B$  does not, i.e. for non-trivial inconsistent worlds, and for worlds where  $C$  holds but neither  $D$  nor  $\sim D$  do, i.e. for nonnull incomplete worlds.<sup>(6)</sup> The classical rule has to be rejected. With only very weak (De Morgan) conditions on negation, e.g.  $\sim(A \wedge B) \leftrightarrow \sim A \vee \sim B$ , the normal (star) rule is inevitable.

To both sum-up and anticipate: the star rule may be variously seen as a generalization of the classical negation rule, as a generalization that is inevitable if inconsistent and incomplete worlds are to be symmetrically allowed for, as deriving from a general analysis of negation as a certain type of one-place connective, as a way of reducing a 4-valued picture to a two-valued one (the American plan to an Australian one), as a natural reversal operation in semantic tableaux and in worlds modellings (all these explications are featured in RLR).

#### 4. HOW THE MAIN THEORIES HAVE BEEN CONFUSED.

There is much confusion of these three different theories in the modern literature, much of it engendered by classical logicians' identification of their negation with "the" real and natural negation. This is responsible for many gratuitous problems. For example, Strawson thinks that he is giving an account of negation which explains its behaviour in modern logical theory, but he offers the cancellation account, and then (correctly) arrives at principles implying Aristotle's thesis which would trivialize the modern theory.

The first and second views (i.e. 1 and 2) are spectacularly confused in Findlay's exposition of Hegel, where neither view is particularly appropriate. For Findlay both gives a self-nullifying account of contradictions and also uncritically assumes without any pause a classical explosion view:

All these doctrines [of Hegel's Dialectic] are extremely hard to stomach, since a contradiction is, for the majority of logical thinkers, a self-nullifying utterance, one that puts forward an assertion and then takes it back in the same breath, and so really says nothing.<sup>(7)</sup> And it can readily be shown that a language system which admits even *one* contradiction among its sentences, is also a system in which *anything whatever* can be proved... (p.76)<sup>(8)</sup>.

The last claim is false, since there are many language systems containing only isolated contradictions; in particular, systems where contradiction really is self-nullifying are commonly of this type. Findlay cannot, on his own grounds, have it both ways.

It is accordingly not surprising that Findlay is bound to say that Hegel, whatever he might *say*, did not mean by 'contradiction' contradiction.

... whatever Hegel may *say* in regard to the presence of contradictions in thought and reality, the sense in which he admits such contradictions is determined by his *use* of the concept and not what he says about it... it is plain that he cannot be using it in the self-cancelling manner that might at first seem plausible. By the presence of 'contradictions' in thought and reality, Hegel plainly means the presence of opposed, antithetical *tendencies*... (p. 77, similarly, p.193).

Such is the myth, which is recounted with minor variations, by the majority of commentators upon Hegel. That was not Hegel's position, as Hegel emphasized. As Findlay himself elsewhere remarks,

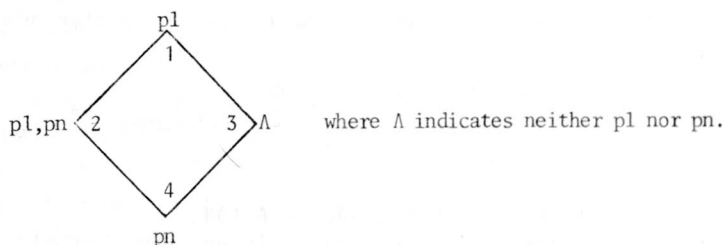
Hegel makes it as plain as possible, that it is not some watered-down equivocal brand of contradiction, but straight-forward head-on contradiction, that he believes to exist in thought and the world... (p.77),

not antithetical or opposed tendencies, etc. To the question of how negation and contradiction did function on Hegel's logic, the dialectic will return.

The confusion is not confined to nonlogicians, but appears to originate in Aristotle. Russell, one of the main architects of classical logic, affords yet another, and striking example. For in *Human Knowledge: Its Scope and Limits*, Russell tries to explain negation: but he outlines (on p.519) a way of introducing negation which leads *not* to classical negation but rather to a relevant negation (which indicates that even some of those who thought they were arguing for classical negation have gone astray). Russell links negation ('not') with 'No'; correspondingly affirmation is linked with 'Yes'. But "Yes" means "Pleasure this way", and "No" "pain that way" according to Russell, whence the correspondences:

Affirmation	Negation
Yes	No
Pleasure, that way (pl)	Pain, that way (pn).

The first thing to observe about such an explanation is that the explaining terms are *neither exclusive nor exhaustive*. For one phenomenon may yield neither pleasure nor pain, another can produce both pleasure and pain. So what Russell's analogy leads to is not a two-valued picture but a four-valued lattice, with the following Hasse diagram:



Since the negation operation,  $N$  say, defined on the lattice (in terms of pl yielding pn and vice versa) plainly takes us just from top and bottom and vice versa, the sides being fixed points, upon representing *and* and *or* in the usual way (in the way Russell invariably took them) as lattice join and meet, what results is a model of relevant logic, specifically of tautological entailment (as presented e.g. in Anderson and Belnap).

For the operations yield at once the following  $\wedge$   $\vee$   $\sim$  matrices

$\wedge$	1	2	3	4	$\vee$	1	2	3	4	$\sim$	$\rightarrow$	1	2	3	4
1	1	2	3	4	1	1	1	1	1	4	1	1	4	4	4
2	2	2	4	4	2	1	2	1	2	2	2	1	1	4	4
3	3	4	3	4	3	1	1	3	3	3	3	1	4	1	4
4	4	4	4	4	4	1	2	3	4	1	4	1	1	1	1

Upon taking 1 as the only designated value (a natural choice since it is the only case of unmixed pleasure, the only clearly desirable element), and upon

defining an implication  $\rightarrow$  by the  $\rightarrow$  matrix above, tautological entailment automatically results.<sup>(9)</sup> For the above, Smiley matrices, are characteristic (see Anderson and Belnap, p.161). Nothing Russell goes on to say alters the initial picture, which he rightly says is 'remote from what the logician means by 'not''. But his attempt to fill the intervening steps, to get to what Russell the logician at least means is simply this:

"not" means something like: "you do right to reject the belief that ...". And "rejection" means, primarily, a movement of aversion. A belief is an impulse towards some action, and the word "not" inhibits this impulse (pp.519-20).

The picture remains four-valued, since the one thing may both impel and repel or may, alternatively, do neither. The four-valued picture is hardly new, going back to the Megarian logicians. Some of the Stoics tried to reduce the truth value picture to a three-valued one, discarding the value, both (true and false).

The valued (matrix) picture may however, like the three-valued picture be reduced to a two-valued worlds picture. The 4-valued matrices can be derived from a semantical model with just two worlds: T, the real world or in this case the pleasure world, and T\*, its reverse, here the pain world. What is of importance for present purposes in this semantical analysis of the matrices is the fact that the negation rule required by the analysis is the star rule, in the form

$$\begin{aligned} I(\sim A, T) &= 1 \text{ iff } I(A, T^*) \neq 1 \text{ and} \\ I(A, T^*) &= 1 \text{ iff } I(A, T) \neq 1, \text{ where of course } T^{**} = T. \end{aligned}$$

## 5. MAIN THEMES CONCERNING TRADITIONAL NEGATION, ORDINARY AND NATURAL NEGATION, AND THEIR MODELS.

Neither the explosion nor the cancellation view is satisfactory. The explosion view is strongly paradoxical,<sup>(10)</sup> the cancellation view is weakly paradoxical (at least as it stands). The cancellation view does not have each contradiction entailing everything, and all inconsistent theories trivialized in the way that the explosion theory does; but it does have each contradiction entailing each other,  $A \wedge \sim A \leftrightarrow B \wedge \sim B$ , for arbitrary A and B. For A and  $\sim A$ , and B and  $\sim B$ , say exactly the same, namely nothing.<sup>(11)</sup> The explosion view is wrong because contradictions are not so destructive: there are various different non-trivial inconsistent theories. The simple cancellation view is also defective, since not all contradictions carry the same information: they differ in what they entail, some of them entailing some things, others other things.

The negation of Hegel's logic, like that of any paraconsistent logic, does not, and cannot, conform to the classical view 2, nor does it conform to view 1.<sup>(12)</sup> For not only did Hegel reject the idea that contradictions could



not be separately thought ('Contradiction is the very moving principle of the world: and it is ridiculous to say that contradiction is unthinkable', *Logic* p. 174; cf. too Findlay, p. 75 where several references are cited: 'it is absurd to say that contradictions are unthinkable'); he also certainly held that in thinking contradictions, one was not thinking nothing, or *merely* a self-cancelling thought; for, quite the contrary, in thinking that Being is identical with Nothingness and is also not identical therewith, one is thinking an explicitly contradictory thought of fundamental importance. While modern paraconsistent theories are usually not as extravagant as to the range, type, or centrality of the contradictions asserted, the intention is much the same: accounts of negation of type 3 are required.

Before considering in more detail what such negations are like, it is worth inquiring, and important to inquire - since classical logic is wont to claim that history (as well as God and Truth and Language) is on its side - what traditional negation, the negation of traditional logic (if there was such a single creature) was like. What was the tradition, especially as regards negation? There wasn't a single unified tradition, there were various competing traditions in particular as to negation and implication. These competing positions are especially evident in the debate as to implication in ancient Alexandria, and in the controversies of scholastic writings. Despite the competition, there seems, at least from post-scholastic times, to have been a dominant view, namely the cancellation view.<sup>(13)</sup> It should be stressed that this is very much a working hypothesis. There is a great deal of difficult assemblage of historical evidence still to be accomplished (both for modern and for earlier periods).<sup>(14)</sup> A weaker theme, on somewhat firmer ground, is that *the mainstream or dominant negation of traditional logic is distinctly nonclassical*. Some of the evidence supporting this first working hypothesis will emerge below.

It is also important to inquire what natural negation, negation of natural language, is like, because part at least of the logical enterprise concerning negation is to reflect key features of that negation. Again it has been assumed, with precious little evidence, that classical negation fulfils this role. Many considerations tell against this assumption (see RLR 2). It is important to see through classical negation's pretensions to be the ordinary normal intuitive notion of natural language and logical thought - compared with which alternative negations such as relevant negation must be seen as 'deviant', 'peculiar', 'queer', abnormal, contrived, or purely formalistic. For seeing through its pretensions is an essential part of seeing through classical (implication) theory and seeing why relevant (implication) theory should replace it.

In fact the situation is pretty much the reverse of the conventional pic-

ture. Relevant negation has a better claim to be the (primary) negation determinate of natural language than classical (if indeed there is a unique natural language negation, which is to be doubted). A second working hypothesis is, then, that *relevant negation is a natural negation*. (A, because the negation determinable is probably the most commonly occurring natural language negation; see further RLR, 2.9).

## 6. NEGATION AS OTHERTHANNESS, AND PROGRESSIVE MODIFICATION OF THE TRADITIONAL PICTURE.

In order to discuss the traditional idea that negation is otherthanness, and to consider negation in its historical setting, especially that of the nineteenth and earlier twentieth century work (when logic began its modern revival), it is helpful to introduce some of the ideas of Boole and Venn in an exact fashion. Consider, in particular a Boole-Venn interpretation of sentential logic S: such an interpretation can be extended to other logics, e.g. modal logics. Such an interpretation  $j$  is a mapping from (initial) wff of S to V which consists of a composite with (at least two) components, e.g. a geometrical area, a set, a mereological class, such that the following conditions are met:

$$j(\sim A) = V - j(A);$$

$$j(A \& B) = j(A) \cap j(B) \text{ i.e. the common part}$$

$$j(A \vee B) = j(A) \cup j(B) \text{ i.e. the union (of areas).}$$

A wff C of S is said to be *BV-valid* iff, for every mapping  $j$ ,  $j(C) = V$ , i.e. the interpretation is always the whole of V. Then no doubt soundness and completeness may be established: a wff C is a theorem of S iff it is BV-valid. Further, assignments under  $j$  may be reduced to assignments for initial wff only, and the conditions given used to definitionally extend the assignments to all wff. What is of especial interest is however not the familiar results but the rendering, or intended interpretation, of function  $j$ . There are at least three overlapping groups of readings:

1. Geometrical reading with  $j(A)$ , or  $|A|$  as it will sometimes be written, as what A covers (cf. Hospers above), or the area (or territory) of A.

2. Set-theoretical readings, with  $j(A)$  some set, e.g. the set of cases where A is true (i.e. the range of A). Set readings are central in the nineteenth century theories of complex terms - in the context of which negation was characteristically discussed. For this the wff of S are reconstrued as complex terms,  $\sim A$  as non-A (e.g. non-animal),  $A \vee B$  as A or B (e.g. animal or plant), etc. Then  $j(A)$  is the extension of A, e.g.  $j(\text{horse})$  comprises (all) horses. While the operations of *and* and *or* are relatively straightforward in

forming complex terms, several logicians were distinctly worried about *not*, and in fact opted, as we shall see, for a non-Boolean interpretation of *not*.

3. Propositional readings, where  $j(A)$  is some proposition or sum of propositions. In particular, an *obvious*  $j$  function, exploited below, is that which maps each initial wff to the proposition it expresses.

The late nineteenth century view was that negation, which applied to terms and also to judgements, is otherthanness, and on the prevailing view restricted otherthanness. Thus according to Baldwin's *Encyclopedia* (p. 147) 'Not-A = other than A - a second thing to A'. But *it was not anything other than A*. Joseph, for one, considers the view that 'whatever it (the positive term) be, the negative term covers everything else', and rejects it. His conclusion is that

A positive term and its corresponding negative (e.g. blue and not-blue) may then be said to divide between them not indeed the whole universe, but the limited universe of things, to which they belong (p. 44fn).<sup>(15)</sup>

Venn himself acknowledges such limits:

It is quite true that we always do recognize a limit, sometimes expressed but more often tacit, as to the extent over which not-X is to be allowed to range; and also we not infrequently do so in respect of X itself; so long as these expressions are set before us in words, and not in symbols only.

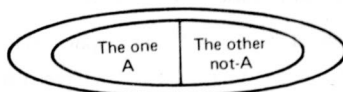
Though he continues, 'Between them X and not-X must fill up the whole field of our logical enquiry', he admits restricted fields, and allows that we can choose the universe (of discourse) - admissions that run him into serious trouble where different negative terms call for different restricted fields. Venn's procedure and the Boolean picture captured in the  $j$ -functions then break down.

It is by now more or less clear how to repair matters. The picture on the left gives way to the picture on the right:

V (perhaps restricted)



V (unrestricted)



In functional terms it is a little more complex, but again now evident enough. A further operation  $*$  is added to the modelling and the rule for negation amended to:

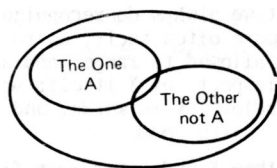
$$j(\sim A) = j^*(A), \text{ i.e. roughly}$$

$$= V_A - j(A), \text{ where } V_A \text{ is the universe as restricted by } A.$$

Some of the intended properties of  $*$  are evident enough, e.g.  $j^{**} = j$ , i.e.  $*$  is involutory, and on British perceptions  $j \cap j^* = \Lambda$ , i.e.  $j$  and  $j^*$  are always exclusive.

To the resulting picture Hegel added, in effect, a significant liberalizing element - one that is forced if contraposition principles (etc.) are to be duly respected, especially at the statement stage - namely that exclusiveness is not always guaranteed. The reason is, according to Hegel, that for certain  $A$  (of categorial type)  $A = \text{not-}A$ . It is not however necessary to accept, for anything that follows, this difficult doctrine: it would be nearer the mark to say that what is supposed is that there are situations where  $A \leftrightarrow \sim A$ , and better to say, still less demanding, where both  $A$  and  $\sim A$  hold. It is enough to say, with Simone de Beauvoir (p.18) that presence and absence are not mutually exclusive, or that what  $A$  covers does not fully exclude what  $\sim A$  covers. The liberalized picture which emerges is important:

$V$  (unrestricted)



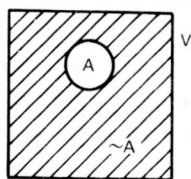
Given that  $j^*$  need not exclude  $j$ , the rule  $j(\sim A) = j^*(A)$  again suffices (equivalently, and revealing more structure,  $j(\sim A) = V - \bar{j}(A)$ , where  $\bar{j}$  is the inverse of  $j$ : see RLR, 13.5). To glance ahead, we shall simply put The One and The Other back to back, as in a phonograph record, and we will have, more or less, the sought picture of negation.

## 7. TRANSPOSING THE HEGELIAN PICTURE: RESTRICTED OTHERTHANNESS, REVERSAL AND OPPOSITES.

The next task is to transpose the whole business (as preclassical thinkers like Joseph also tried to do) from the term to the statement level. The Hegelian picture goes over intact, and what results interpretationally are functions extended not according to Boolean but according to De Morgan lattice logic (for details see Anderson and Belnap, or RLR). The negation is no longer classical, but relevant.

In terms of relevant negation we can see classical negation as a depauperate one-dimensional notion, which forces us to consider *otherness* with respect to a single universe consisting of everything. In classical logic negation,  $\sim A$ ,

is interpreted as the universe without  $|A|$ , everything in the universe other than what  $A$  covers, as reflected in the Venn diagram:



The square  $V$  comprises  
the universe

The universe can be interpreted as the sum of propositions. Thus where atomic wff  $p$  is interpreted, naturally enough, as the proposition it expresses,  $\sim p$  amounts to every proposition in the universe other than the proposition that  $p$ .

Relevance problems come straight out of this; for irrelevance is written in at the bottom. All contradictions have the same interpretation, namely  $V$ : hence each entails all others and indeed everything. Paradoxes are inevitable.

It is corollary that  $\sim p$  cannot be independently identified, it is entirely dependent on  $p$ . This relates, more than coincidentally, to alienation (compare what Simone de Beauvoir has to say to alienation of women where 'woman' is identified as 'other than man'; and is not positively identified, only introduced as alien to the primary notion, 'man'). The negation  $\sim A$  of  $A$  is (so to say) alien to  $A$ .

Relevant negation can, however, preserve much of the *otherness* notion of traditional negation (without the counterproductive alienation features). But relevant and classical negation differ firstly as regards what the otherness is considered in relation to. In the case of classical negation it is otherness with respect to the universe. In the case of relevant negation it is otherness with respect to a much more restricted state, such that  $p$  and its negation do not (interpretationally) exhaust the universe between them.

Such a restricted otherness notion is provided by *reversal*, which gives the other side of something. The *lead* side and the *other*, or opposite, side do not yield everything, the universe, by any means,<sup>(16)</sup> any more than  $p$  and  $\sim p$  yield everything with relevant negation. Reversal is in fact a restricted *other than* notion - on the other side is not all territory other than  $p$ , representing everything other than  $p$ . With reversal otherthanness operates in a relevantly-restricted universe. The reverse direction (or sense) is not *any* direction other than the forward or given one.

The reversal picture can be filled out in several apposite (and of course connected) ways, both more superficially syntactically, since in one sense the reverse of  $p$  is  $\sim p$ , and less superficially semantically. Consider first the *debate*, or *dialectical*,<sup>(17)</sup> *model* which reveals the type of restricted situa-

tion with respect to which otherness (the rest of the situation) is assessed. A debate can be represented as the  $p$ -issue, or the  $p$ -question, when the issue is as to whether  $p$  or  $\sim p$ . One side asserts, argues, or defends  $p$ , the other side  $\sim p$ . Or, as we say,  $p$  and  $\sim p$  are each sides of the issue as to whether  $p$ , one side being the opposite (X or reverse) of the other. The sides are clearly *issue-restricted*, and so accordingly is the complementation. To present the case for one side, e.g. the positive or affirmative, and to present the case for the other side, the negative, is *not* to present the case for everything, to exhaust what can be said, etc.

The debate model indicates that classical negation itself carries the seeds of irrelevance. Thus if one is debating an issue, whether  $p$  or  $\sim p$ , classical negation would allow anything at all that wasn't  $p$  as relevant to truth of one half. Thus in debating say, uranium mining one could introduce say, child care centres as relevant to one side of case. The notion of relevance is similarly destroyed, since anything confirming anything which is not  $p$  is relevant to the debate. Notions of aboutness, of case, issue, relevance, confirmation and evidence, are all seriously distorted, in a systematic way, by classical negation (as independently shown in much detail in RLR and [22]). The systematic distortion is a result of the restriction to (complete) possible (consistently describable) worlds, a restriction forced by retention of classical negation. There is a similar, and similarly forced, distortion of other intensional functors, e.g. of deontic functors such as obligation (with respect to moral conflicts), of psychological functors such as belief (with respect to inconsistent beliefs), etc. etc.

Classical negation is a depauperate one-dimensional concept which distorts the functions of natural language and limits the usefulness of the logic it yields. Classical negation may *seem* natural, firstly because we (or rather some, the brainwashed among us) have become accustomed to it and perhaps impressed by its computer applications and arithmetical analogues, and secondly because (like material implication itself) it captures *one* dimension of negation, but it has rejected the other dimensions (e.g. restrictedness). Classical negation gives a simple account which is a limiting case, but one which, like that of frictionless surface or perfectly elastic body, does not occur in experience.

## 8. SEMANTICAL MODELS: WORLDS ON RECORD AND TAPE;

The debate model can be given a more semantical turn. In the  $p$ -issue,  $\sim p$  is asserted, or presented as true, on one side, a say (i.e.  $a \models \sim p$  in obvious notation), while the reverse, namely  $p$ , is asserted, or presented as true, on the opposite side  $a^*$  (i.e. symbolically  $a^* \models p$ ). Now one side succeeds in a

debate, or establishes its case, iff the opposite side does not; therefore  $a \models \sim p$  iff  $a^* \not\models p$ . That is, a version of the star rule naturally emerges from the debate model more semantically considered. Statement  $\sim p$  is made, or presented as, true at side or situation  $a$  iff  $p$  is made, or presented as, true at its opposite  $a^*$ .

The debate model leads directly to the *record cabinet model*. The cabinet, which can represent the files of the universe, is full of records, each record is an *issue*, or question, with  $p$  on one side and  $\sim p$  on the other side, for every atomic  $p$  (at least). From this point of view classical negation takes  $p$  as one side of one record, and  $\sim p$  as everything else in the cabinet (classical theory fails to duly separate issues). Relevant negation takes  $p$  as one side of the record and  $\sim p$  as the other side of the *same* record, there being many many records in the cabinet. Note well that intensional functions select a program from the cabinet. Such a program may include *both* sides of a record, and may include neither side of various records - in contrast to the published classical picture (the classical picture can be suped-up to avoid the latter defect but not the former).

The cabinet model may be differently oriented. Each record, or tape, represents, e.g. it may just *describe*, a world, a two-sided world. Then where  $a$  is one side of a world record, or a world, the opposite side is again  $a^*$ , where  $*$  is the *reversal*, or *flip*, function which gives, whichever side one is in on, the other side. Obviously  $a^{**} = a$ , since turning the record over twice takes one back to the initial position. The semantical rule for evaluating negated statements is, as for the debate model, the star rule,  $\sim p$  holds at  $a$  iff  $p$  does not hold at  $a^*$ . By contrast, the classical rule quite erroneously identifies a side with its opposite.

The records may be ordered or arranged in a way that reflects the relational structure of (two sided) worlds. The *structured record model* corresponds exactly to a natural elaboration of Kripke's valuable sheets-of-paper model of semantic tableaux for normal modal logics. In explaining *alternative sets* Kripke says (63, p.73): 'Informally speaking, if the original ordered set is diagrammed structurally on a sheet of paper, we copy over the entire diagram twice, in one case putting in addition  $A$  in the right column of tableau  $t$  and in the other case putting  $B$ ; the two new sheets correspond to the two new alternative sets'. Thus a full construction which consists of a system of alternative sets corresponds to an arrangement of sheets (a sheaf of sheets). For relevant semantic tableaux there are only two innovations. First, whereas with strict implication new related tableaux are introduced one at a time, with relevant implication new related tableaux are introduced two at a time, i.e. in pairs. This reflects the replacement of the two-place alternativeness

relation of modal logics, by the three-place alternatives relation of relevant logics. The first innovation is not particularly germane to the present issues (and quasi-relevant systems such as the I systems which require only two-place relations could be adopted for exposition). Second, and more important, then, both sides of the sheets are used. (Relevant logics are conservation-oriented in that even if rather a lot of sheets are introduced, both sides are used; the reverses are not wasted as with modal semantical tableaux). The reversal function \* accordingly reverses the page, giving back for front and front for back.

In sum, reversal and opposition have the right properties in leading respects for (the semantics of) relevant negation. Thus the opposite side of something is not the removal of the first side or, for example, everything other than the first side; it is *another* and *further* side, which is relatively independent of its reverse but which is related to it in a certain way. Both sides can co-occur (occur simultaneously) in a framework (e.g. controversy) and one can perfectly well consider both of them. The important point, to say it yet again, is that one side does not somehow *obliterate* or wipe out or entirely exclude or exhaust its opposite. Nor is the reverse, or opposite, just defined negatively as *the other* - it has an independent and equal role on its own behalf.

There is no mystery then about relevant negation. It is an otherthanness notion; it has natural and easy reversal models. There is some mystery however about classical negation, except as an extrapolation, and much mystery as to why some logicians are tempted to apply it everywhere, especially where, as so often, it mucks things up. Indeed, given the naturalness of relevant negation as issue-controlled complementation, versus the unnaturalness of classical; the naturalness of the reversal notion; and the improved ability of relevant negation to account for actual intensional functions in natural languages, relevant negation has a far better claim to be considered the core negation relation of natural language than classical. So much for the classical claim to have the only real natural negation and that relevant negation is queer.

#### APPENDIX 1. HISTORICAL SIDELIGHTS; NEGATION AND CONTRADICTION IN HEGEL AND HEGELIAN TRADITIONS.

There is not in Hegel a complete and well worked out theory of negation. There is however much that is suggestive, many models, and a clear nonclassical paraconsistent view. According to Hegel, contradiction occurs both in thought and in the world. There are true contradictions in nature, as an analysis of motion shows.



Something moves, not because it is here at one point of time and there at another, but because at one and the same point of time it is here and not here, and in this here both is and is not ([12], II, p.67).

For details Hegel refers to Zeno's paradoxes of motion. Another important class of true contradictions concerns the categories, which can pass into and be identical with their opposites. Representing propositional identity as a coentailment, there are truths of the form  $A \leftrightarrow \sim A$ . Hegel nicely contrasts his view with the ordinary view:

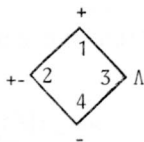
Ordinarily... contradiction, both in actuality and in thinking reflection, is considered an accident, a kind of abnormality or paroxysm of sickness that will soon pass away ([12], II, p.67).

Ordinarily too, often enough, contradictions are considered unthinkable. According to Hegel however,

The only correct thing in that statement (that contradiction is unthinkable) is that contradiction does not end the matter, but cancels itself. But contradiction, when cancelled, does not leave abstract identity; for that is only one side of the contrariety ([13], p.174).

The other side is presumably difference. Although there are elements of a cancellation picture both here and elsewhere in Hegel (e.g. [13], p.172 where he compares positive and negative with + and -, which cancel to zero), he rejects a cancellation view. He specifically notes ([12], p.59) that *the ordinary view of contradiction is that it reduces to nothing*, like a vacuum (in itself a revealing piece of historical data). But he says ([12], p.70) that we must pass beyond this one-sided resolution and 'perceive its positive side, when it becomes absolute activity and absolute Ground'.

As well as a severely qualified concellation picture, Hegel offers us a polarity picture of negation, drawn from physics, with Positive and Negative as polar opposites ([13], p.174). This polarity picture rapidly leads to a four-valued model. For some things are both positive and negative, and others are neither. In short we are back with the lattice Russell's theory leads to:



Havas also claims to find a many-valued logic in Hegel, though what his evidence is is unclear;

... in Hegel's view, in addition to the values "true" and "false", there is another value, namely, "true and false" and this is the designated value. So, in this case, the value "not-true" is not identical with the value "false",

since "not-true" means "false, or true and false". If a proposition does not have the value "true", it will have either the value "false" or the value "true and false". Propositions having the value "true and false" are expressions of the actual being of the things, that is, their existence in the dynamical states of coming into being and passing away, and not the mere subsistence of the things.

Perhaps the most disconcerting things about Hegel's logic are firstly, that there appears to be no distinction between acceptable and unacceptable contradictions, all being in a way unacceptable, and generating by themselves motion towards a higher stage in which they are partially resolved, and secondly the sheer extent of contradiction: '*All things are contradictory in themselves*' ([12], p.66). In later idealists such as Bradley it was insisted that contradictions were manifest in appearance, but not in reality; the Absolute was claimed to be self-consistent. Hegel's view seems to have been different; the Absolute was inconsistent: 'the sum-total of all realities ends as absolute contradiction' ([12], p.69).

It is perhaps because of the difficulty of admitting contradictions on such a grand scale as Hegel does that the Marxist tradition, while retaining the thesis of contradictions as pervasive and the source of all movement, watered down the notion of contradiction.

In Marxist theory, the notion of contradiction degenerates - exceedingly low redefinitions of 'contradiction' are invoked. This degenerating use of 'contradiction' which is already beginning in Marx's work has become highly advanced in modern Marxism, where 'contradiction' comes to mean simply 'problem' or '(apparent) conflict' (as often in Mao) or even 'difficulty' (in an Australian radio broadcast). Just one example from *The Trojan Horse*:

So far no major breakthrough has appeared that is capable of resolving the contradiction of uneven regional development (p. 183, similarly p. 182).

Here the 'contradiction' involves no inconsistency; what there is rather is a problem which has not been satisfactorily resolved.

## APPENDIX 2. AN ACCOUNT OF NEGATION AND CONTRADICTION IN WITTGENSTEIN'S WORK.

In the earlier work, the *Notebooks* and especially the *Tractatus*, Wittgenstein runs together, in a way that is ultimately incoherent, exclusion and cancellation models of negation. On the one hand, a classical explosion model of negation and classical truth tables for negation are adopted; negation is represented as total exclusion. There has, of course, to be more to the account of negation than this. In particular, logical constants such as negation, since

they would otherwise raise serious difficulties for the picture theory of meaning, call for special treatment, which they obtain through the theory of truth-functions. Negation is simply such a classical function; nothing in reality corresponds to it.

But, on the other hand, significant elements of a cancellation picture are superimposed on the classical view. Although the simple parts of contradictions *and* tautologies have sense, 'the connexions between these paralyse or destroy one another' ([35], p.117). 'Tautology and contradiction are the limiting cases - indeed the disintegration- of the combination of signs' ([34], 4.466). Wittgenstein even says explicitly that 'in a tautology the conditions... cancel one another...' ([34], 4.462), *but* 'cancel' is applied in a different context from that where the cancellation view is explained. (What are cancelled, according to Wittgenstein, are 'the conditions of agreement with the world'). The expected corollaries of a cancellation picture follow, but for tautologies as well as contradictions: they 'say nothing' (e.g. [34], 6.11)<sup>(18)</sup>.

In later work this unstable combination of a hard classical view with a cancellation picture is modified and softened in several respects:

1. Negation is not one thing, or one function. Wittgenstein rejects 'the idea [of earlier work] that there is something common to all negation..., that negation always has "the same meaning" ' ([9], p.540, where these claims are referenced). Wittgenstein now wants to insist that the meaning of negation is not an object, and not an essence. Rather, what the meaning of negation is is shown by 'the way it *works* - the way it is used in the game' ([31], p.55). The exclusion model is not abandoned: rather it is assimilated as one among many (partial) models of negation. So it is also with contradiction (which, along with negation, receives considerable attention in Wittgenstein's later work):

... contradiction isn't the *unique* thing people think it is.  
It isn't the *only* logically inadmissible form and it is,  
under certain circumstances, admissible (Letters, p.177) .

The no-one-thing theme concerning negation (and derivatively, contradiction) can evidently be assimilated under the determinable theory of negation (of [8], 4.3 and RLR 2.9). There is not a single negation determinate - and in this sense, no essence - but *many*, with classical and connexive negations as (depauperate, and hardly ideal) limiting cases. The determinable theory handles well Wittgenstein's comparison of  $\sim(\sim p) = p$  with  $(\sim\sim)p = \sim p$ , where he says both that the meaning of negation is not different and that there is some truth in our inclination to say that ' $\sim$ ' must mean something different in the two cases ([36], p.81).

2. Much of the remainder of Wittgenstein's apparently diffuse, and some-

times incoherent, material on negation and contradiction can similarly be coherently organized within a wide relevant theory of negation, a theory which has a classical core but allows for a wide variety of nonclassical language-games or situations. *The key correspondence* in so reorganizing *is that between language-games* on the one side *and situations or worlds or - a bit differently - theories* on the other. Language-games can play a quite analogous semantical role to that played by situations in world semantics (and pragmatics), and indeed, semantical analyses in terms of worlds can be recast in terms of games<sup>(19)</sup>.

Among the normal worlds, comprising class K, of relevant logic, there is a distinguished world T, the factual world, which (on more orthodox accounts) is exclusive and exhaustive, i.e. the classical negation rule is satisfied at T, as it is at the subclass P of K, the complete possible worlds (of modal theory). P does not exhaust K, since K also contains incomplete worlds, which may or may not be possible, and inconsistent worlds. Normal worlds do not of course exhaust worlds (i.e., class W), they only exhaust the worlds required for the assessment of such notions as deducibility and entailment; but for semantical assessment of more highly intensional functors *and* of connexive logic, abnormal worlds are also needed. A precisely analogous picture holds good, in principle<sup>(20)</sup>, for language-games; and we shall simply indicate corresponding languages-games and classes of such games by bar superscripting of corresponding worlds. Thus, for instance, corresponding to T is the true-false language-game  $\bar{T}$ . Also, most important, corresponding to inconsistent and very incomplete (and so nontrivial) theories in K are fragmentary language-games of  $\bar{K}$  in the form of inconsistent calculi (Wittgenstein equates calculi with language-games). Through the difference between  $\bar{T}$  and other elements of  $\bar{K}$  we can account for such facts as that Wittgenstein does not (in the *Lectures*, for instance) really get beyond a classical truth-table account of negation (good for elements of  $\bar{P}$ ), yet says enough to make it clear that that is inadequate, because we can allow contradictions in systems (cf. [36], p.138) and not admit that everything follows (cf. p.243); so contradictions which occur need not trivialise a calculus in the way that they do classically.

The distinction between  $\bar{T}$  and inconsistent calculi in  $\bar{K}$  appears in (unnecessarily) accentuated form in Wittgenstein's transitional work (especially [20]). There Wittgenstein separates pure calculi where discourse is not really propositional from the true-false game, which does involve semantical matters such as truth and falsity, and where discourse is propositional<sup>(21)</sup>. He even seems inclined to suggest that, contrary to appearances, a single notion of contradiction does not bridge these distinct areas. Contradiction proper is propositional; 'The idea of contradiction... is that of logical contradiction, and this can occur only in the *true-false game*, that is where we make statements' ([20], p.126). What is *forbidden* in calculi, certain configurations

such as Hilbert's  $0 \neq 0$ , sometimes called 'contradictions', should be represented by 'entirely new sign(s)... the sign Z, say' ([20], pp.175-6). This doubtful distinction is in danger of disappearing even where it occurs (as Waissman's puzzlement about Wittgenstein's position in [20] helps reveal). For Wittgenstein also emphasizes that 'a contradiction proper and a tautology do not *say* anything' ([20], p.531), i.e. even in the true-false game contradictions have no content, a claim Wittgenstein often equates with the claims he also makes that they have no sense, and do not express propositions, but are only sentences (cf. [36], pp.185-6). In this way a propositional role for contradictions which distinguishes them from symbols like  $0 \neq 0$  in perhaps inconsistent calculi is undermined. Of course a residual distinction could be retained by saying that in one area the signs are associated with propositions (e.g. their parts express in them), in the other area not. But it would not be worth much. And in any case the effect of such a nonpropositional line on logic, and elsewhere, is devastating (unless a *great* deal of implausible reinterpretation of propositions and proofs in terms of rules is undertaken). For example, things that figure in proofs, arguments (such as from  $p$  and  $\sim p$ ), beliefs, etc., can no longer figure because they are not duly propositional. In particular, proofs using *reductio* methods are fouled up.

In later work, Wittgenstein abandoned the distinction between types of "contradiction" (but by no means entirely the theme that contradictions do not make sense), and speaks of configurations in calculi as contradictions. It is calculi with such configurations (members of  $\bar{W}\text{-}\bar{P}$ ), and their mathematical investigation that afford the *limited* emancipation from the requirement of consistency predicted by Wittgenstein ([32], p.332). There is said to be point in developing systems in which contradictions can be generated ([36], pp.188-9), and situations, language-games, where there is such point are outlined.

There are then different (sorts of) language-games with respect to contradictions, some such as  $\bar{T}$  where they would be very damaging if they occurred, and some such as calculi games where they do occur but without damage. By way of different language-games, different procedures in the face of contradictions can be allowed for. Thus around the theme of different language-games, many of the pictures and suggestions tried out, often rather inconclusively, in Wittgenstein's later work, can be organized. The reversal pictures of negation (cf. [36], p.180), for example, fit immediately into the wider relevant framework.

Among the pictures tried out are elements that echo Hegel, for instance the sickness presentation of contradictions, the rejection (late on, e.g. [32], p. 130) of the unthinkability of contradictions. Wittgenstein was also aware of, and not entirely unsympathetic to, the Hegel-Parmenidean thesis that an adequate description of motion involves contradictions ([32], V). But Hegel's progressive transcendence of contradictions (which are supposed to appear almost

everywhere) is not repeated in Wittgenstein, who would, for the most part, have us stop at contradictions or carefully skirt around them. The sickness presentation is filled out in the *germ* (but not a disease) *image* of a contradiction ([36], pp.158, 211). A contradiction is like a germ in a system, but it does not show that the whole system is diseased. In short, a contradiction in a system is generally a bad thing and to be avoided, but does not reveal triviality. In this connection we are offered something like Kriesel's story regarding Wittgenstein on contradiction: - according to Kriesel, Wittgenstein's rule is: On encountering a contradiction, Stop! There are manifold troubles with this connective-style rule (brought out in [22], pp.179-80), and it should be rejected. Nor it is so clear that Wittgenstein would have accepted it. What Wittgenstein does say concerning finding a contradiction in a system is that 'the contradiction does not even falsify anything. Let it lie. Do not go there' ([36], p.138). This approach is unsatisfactory. For - contrary to Wittgenstein, who assumes that usefulness implies no contradictions ([32], p.104), that where a calculus has a use contradiction has to be forestalled (cf. [9], p.272) - we may want to *use* the contradiction to show significant things about a system. And we *won't* want to persist with a system which proves trivial (as Wittgenstein occasionally admits, [36], p.243).

3. A cancellation view, strictly incompatible with the classical theory of the *Tractatus*, is increasingly infiltrated in subsequent work. A cancellation picture is already much deployed in transitional work, e.g. 'the rules of Euclidean geometry don't contradict one another, i.e. no rule occurs which cancels out an earlier one ( $p$  and  $\neg p$ )...' ([31], p.345). It is such a cancellation picture, where (as in the *Tractatus*) contradictions have no content and say nothing, and so are useless, that lies behind Wittgenstein's assumptions that one should not draw *any* conclusions from a contradiction ([36], p.220), or better, that a way should be found of not proceeding from a contradiction ([36], p.223). But both assumptions are inadequate, because often one needs, or wants, to proceed from a contradiction: some contradictions are very useful.

Very many of the pictures and images of negation Wittgenstein later considers are of a cancellation type or can be adjusted to fit a cancellation model. Although Wittgenstein repeatedly alludes to such images, at the same time he depreciates them (e.g. all attempts to explain why a contradiction "won't work" are spurious, [36], p.xviii): they are all said to convert to no more than substitution of one symbolism for another. Even so, such things can have an explanatory and modelling role. Wittgenstein suggests not, because all that is offered is symbolism and figure, so the question of 'how one is going to use it?' ([36], p.181) remains, since any picture can be used in several ways. He goes on to advance the even more dubious description theme that 'anything

which we give and conceive to be an explanation of *why* a contradiction does not work is always just another way of saying that we do not want it to work' ([36], p.187).

The assumption that contradictions don't or won't, work and associated themes, e.g. that contradictions are useless, and associated images, etc. the jamming picture ([36], pp.187-9, ascribed to Moore, p.190), are all connexivist in cast. With a contradiction, as when the cogs jam, nothing emerges, 'we cannot do anything with it' ([36], p.191). It is from the same cancellation model that the no-content thesis, which jamming depicts, derives that contradictions do not say anything, a thesis also equivalently (but misleadingly) expressed in 'contradictions don't make sense'.

The cancellation view can be included in the relevant synthesis by appeal to abnormal worlds or language-games, games where contradictions do stop proceedings, and where  $A \wedge \neg A$  may have no content. But in assuming, as he often appears to, that games are restricted to those that are classical (effectively, in  $\bar{P}$ ) or those that are of a cancellation type (in a subclass of  $\bar{W}-\bar{K}$ ), Wittgenstein much too drastically delimits the games, or worlds, needed in giving a full account of negation. And in assuming that abnormal cancellation-type games are characteristic - 'that we exclude the contradiction and don't normally give it a meaning is characteristic of our whole use of language' ([36], p.179) - Wittgenstein goes curiously astray. Commonly we do not treat contradictions in this way. We reason on the basis of them (e.g. in reductio arguments), we act on the basis of inconsistent information (cf. the general who acts, and succeeds, on the basis of contradictory reports [36], p.105), we exploit paradoxes when we can, etc.

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## NOTES.

- (1) Joseph fails to escape the difficulty though he makes two attempts, contending (falsely): 1- that 'there is always a positive character as ground of a negation', 2- 'that A is not B means that it is different from B and not that it is non-existent'. Both routes have since been followed through, and found wanting.
- (2) Sartre is firmly entrenched (like the early Russell) in a levels-of-existence doctrine (see, e.g., p.7, middle).
- (3) This assumes - what is not unreasonable, but strictly calls for further argument - that the inner and outer negations are the same.
- (4) Assumed, for the time being, at least, is a metatheory for the semantics that can be interpreted classically: cf. RLR, 3.2.
- (4.1) Quine's position on "deviant" logics is inconsistent. As Gochet points out (in chapter 7, Section 7), Quine has vacillated between a liberal position, according to which every statement, including any logical law, is open to revision, and an incompatible conservative position (reflected in the previous quotation from p.81) where revision becomes im-

possible, any change in logical principles being ascribed to changes in the meanings of constants. Quine's later attempt to resolve the matter (in *Roots of Reference*) and admit *limited* revisability, not only leads beyond the confines of truth-functional logic (to three-valued and indeed intensional logic), but is, so Gochet argues, untenable.

- (5) Where the classical account is not the crude (but persistent) view that the material-conditional represents 'if ... then', it is a classical-based modal theory in the fashion of Stalnaker and others (see, especially, Harper, Pearce and Stalnaker). Both types are criticised in RLR, where the rudiments of an alternative relevant account are also presented. The philosophical basis of the relevant account is explained in Routley (unpublished).
- (6) There are independent arguments, presented in RLR, 2, that  $\wedge$  and  $\vee$  semantic-al rules do not change from orthodoxy.
- (7) Similarly below, 'contradictions... condemn (us to) wholesale dumbness', 'it is the mark of a self-contradictory utterance that it describes nothing whatever', etc. Compare also p.25 where self-nullifying is explained in terms of taking back what has been put forward so as to leave nothing standing.
- (8) 'So', Findlay continues, p.76,  
     "the *whole* of such a system becomes self-nullifying, and infected with contradiction".  
     The attempt to connect the different cases by the use of 'self-nullifying' fails: for a system is self-nullifying, or rather self-defeating, in saying too much, in being trivial, whereas an utterance is self-nullifying in saying too little, zero.
- (9) Alternatively, on different grounds both 1 and 2 may be designated, and a different implication matrix adopted with the same result: see RLR, 2.
- (10) That is only symptomatic of the range of things that is wrong with it, on which see RLR, 1.
- (11) Strictly there are different positions here depending on whether contradictions are said to imply themselves or not, i.e. whether  $A \rightarrow A$  holds quite generally or not. If not, as with peripatetic logics, weak paradox can be avoided. But then many other problems arise: see RLR, 11. An alternative, sometimes attributed to Wittgenstein, is to say that contradictions lead nowhere, that all argument stops when a contradiction is encountered. As to how unsatisfactory this view is, see [22] pp. 179-80. Moreover, on Wittgenstein's view, contradictions may stand in some language games. They are not always destructive or self-cancelling.
- (12) The theme that Hegel's logical theory is a paraconsistent one will be argued elsewhere, as will the theme that contradictions in Hegel's theory are genuine contradictions.
- (13) Where does Hegel fit in? Hegel seems to have realized that he was doing something different from traditional logic, that he was in a sense outside of (and extending) the tradition.
- (14) We should be grateful to anyone who supplies historical leads to pursue. The situation is much complicated in the case of scholastic logicians by the selection of work that has so far been made available - which typically tries to see these people as anticipating modern established doctrine, the conventional classical wisdom, rather than as investigators of various alternative logic options. The bias of history impedes research of alternatives, so to say. Fortunately that situation may be beginning to change especially with new research into the *obligationes*-literature.
- (15) Joseph elaborates his view through examples such as 'intemperate', 'uneven' and 'not-blue', e.g. the latter is equated with 'coloured in some way other than blue' (*italics added*). More generally, not-A will signify what a subject, *which might be A*, will be if it is not A' (p.43).



- (16) Otherwise there would be room for only one record company, and only one record from it.
- (17) In one of the historical senses of 'dialectical'. A debate can also be 'dialectical' in the other historical sense; for one side may defend both  $q$  and  $\sim q$ . A related model is the *evidence model*, where one side is the evidence for  $p$ , the other the evidence for  $\sim p$ .
- (18) It is possible to define a (highly artificial) notion of content which makes *some* elements of such combinations of classical and cancellation views work, for instance thus: where  $A$  is analytic  $ct(A) = 0$ , and where  $A$  is not analytic  $ct(A)$  is defined in a standard way, e.g. in terms of consequences of  $A$ , or through the class situations where  $A$  does not hold. But then content loses its usual (normic) connections, e.g. the ties with consequence are severed, and the logical behaviour of content becomes highly irregular.  
L. Goldstein persuaded us that some of Wittgenstein's early work involved a cancellation view.
- (19) As explained in detail in [8], 7.2. In effect the correspondence is also applied, in different ways, by Lorentzen and Hintikka, where the analogies with game theory are also exploited.
- (20) There are some (hardly insuperable) problems in describing maximally consistent language-games.
- (21) This sharp contrast, and double role for contradictions, is repeated in Hallett (p.221), again based partly on transitional work. Where propositions *say* something, describe something ([20], p.106), a contradiction is alarming. 'For there can be no contradiction in reality (i.e.T) our description must be wrong'. By contrast, contradictions need not be alarming in mathematics (in K-P). 'But mathematics is always a machine, a calculus. The calculus describes nothing. It can be applied to that to which it can be applied'.

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